approximation and such terms are not known, hence for consistency those two terms are not included.

In figure 1 we have have plotted the product of the excitation probabilities and the impact parameter p against p at the incident energy 1412 *keV*.

In figure 2 we have compared the cross-sections obtained from first Born distortion and second Born approximation for $1s$–$2s$ transition. It is seen that at about 1000 *keV* incident energy, the cross-section due to 1st Born, 2nd Born and the distortion approximation are almost the same. As the energy decreases, the discrepancy between the results of the 1st Born and the distortion increases, where the second Born results are comparatively close to the distortion results.

Figure 3 shows our results of the cross-section of $1s$–$2s$ and $1s$–$3s$ excitations. It is found that $1s$–$3s$ excitation cross-section is much lower than that of $1s$–$2s$ excitation.

This work is prelimmary to a series of investigations of Alpha-Hydrogen Collision where we propose to take into account the capture state of He^+ as well.

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On parameter estimation of Gibbs' canonical distribution

By C. G. Chakrabarti

Department of Mathematics, F. C. College, Diamond Harbour, West Bengal.

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In this note first considering Gibbs' canonical distribution as a simple statistical distribution, the fluctuation of temperature has been obtained using the asymptotic properties of maximum-likelihood estimate. Secondly, Einstein's formula for the fluctuation probability has been derived purely from statistical consideration.

After Gibbs, we consider a system in equilibrium as a small part of a large system exchanging its energy with its environment (heat bath). In this case the

system may be considered as a sample and let n (assumed sufficiently large) be the number of components (molecules etc) having the energies ϵ_i $(i = 1, 2, \ldots n)$. For the development of statistical theory each energy-component is taken to obey the exponential distribution as

$$dF(\epsilon, \theta) = e^{-\theta\epsilon} d\Omega(\epsilon) \,|\, \phi(\theta) \tag{1}$$

where $\phi(\theta) = \Sigma e^{-\theta\epsilon} d\Omega(\epsilon)$ and $\Omega(\epsilon)$ is a suitable positive measure. In usual statistical mechanics (Khinchin, 1949) $d\Omega(\epsilon)$ is the number of microscopic states of the system of energy lying between ϵ and $\epsilon + d\epsilon$ and $\Omega(\epsilon)$ is usually called structure function.

The likelihood function L for the system (sample) is given by

$$L(\epsilon_1, \epsilon_2, \ldots \epsilon_\lambda\,;\, \theta) = e^{-\theta \sum_{i=1}^{n} \epsilon_i} \Omega\Big(\sum_{i=1} \epsilon_i\Big) \,|\, \Phi_u(\theta) \tag{2}$$

Where $\Phi_n(\theta) = \phi_1(\theta)\phi_2(\theta)\ldots\phi_n(\theta)$ and the maximum-likelihood estimate of θ is given by

$$\frac{\partial \log L}{\partial \theta} = 0 \quad \text{or} \quad \bar{\epsilon} + \frac{d}{d\theta} \log \Phi_n(\theta) = 0 \tag{2a}$$

Let $\hat{\theta}$ be the maximum-likelihood estimate of θ which is thus taken to be the parameter corresponding to the equilibrium state of the system.

$F(\epsilon, \theta)$ being regular (Wilks, 1962) and equation $(2a)$ having a unique solution (Khinchin 1949), the maximum-likelihood estimate $\hat{\theta}$ of θ for large values of n has a distribution which is asymptotically normal $N[\theta, 1/nB^2(\theta, \theta)]$ (Wilks 1962). Thus for large n,

$$\text{var}\,(\hat{\theta}) = \frac{1}{nB^2(\theta, \theta)} = \frac{1}{\left(\dfrac{\partial \log L}{\partial \theta}\right)^2} \tag{3}$$

Where $\left(\dfrac{\partial \log L}{\partial \theta}\right)^2 = H(\theta)$ is the Fisher's information contained in the sample.

As a system in Statistical Physics contains a large number of components, the appropriate formula for determination of fluctuation of parameter $\hat{\theta}$ is given by

$$\text{var}\,(\hat{\theta}) = 1 \Big| \left(\frac{\partial \log L}{\partial \theta}\right)^2 = 1/\text{var}\,(E)$$

where E is the total energy of the system.

On writing $\hat{\theta} = 1/kT$, we have,

$$\text{var}(T) = \left(\frac{dT}{d\hat{\theta}}\right)^2 \text{var}(\hat{\theta}) = \frac{k^2T^4}{\text{var}(E)} = \frac{kT^2}{c_v} \tag{4}$$

which is of the same form as that for the fluctuation of temperature given by Landau and Lifshitz (1958). Thus this T may be identified with the temperature.

Let now there be a spontaneous transition of the equilibrium state corresponding to the parametric value $\hat{\theta}$ (hereafter denoted by θ) to a neighbouring non-equilibrium state corresponding to the parametric value $\theta+\Delta\theta$. Let us now introduce a measure of divergence (or distance) between the two states of equilibrium and non-equilibrium, according to Rao (1945)

$$\Delta s^2 = H(\theta)\Delta\theta^2 = nB^2(\theta, \theta)\Delta\theta^2 \tag{5}$$

But $\Delta s = \sqrt{n}B(\theta, \theta)\Delta\theta$ is normal $N(0, 1)$ for large n (Wilks 1962) Thus we may define the fluctuation probability by

$$P(\Delta s) = \omega\alpha\, e^{-\frac{1}{2}\Delta s^2}$$

Since

$$\tfrac{1}{2}\Delta s^2 = \tfrac{1}{2}H(\theta)\Delta\theta^2 = \overline{\log L(,E\ \theta)} - \overline{\log L(E, \theta+\Delta\theta)}$$

(Kullback 1959) if S and $S+\Delta S$ be the thermodynamic entropies of the states corresponding to the parametric values θ and $\theta+\Delta\theta$, then $\Delta S = \frac{1}{2}kH(\theta)\Delta\theta^2$ gives the entropy difference between the equilibrium and non-equilibrium states. So the thermodynamic fluctuation probability becomes

$$\omega = \omega_0 e^{\alpha} \tag{6}$$

which is Einstein's formula for the fluctuation probability.

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